

Cosmological constraints on the curvaton web parameters

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We consider the mixed inflaton-curvaton model in which quantum fluctuations of the curvaton field during inflation lead to a relatively large curvature perturbation spectrum at small scales. The predicted spectrum is characterized by the large positive tilt and the non-Gaussianity of the perturbations is also large. We obtained the constraints on the model parameters considering the process of primordial black hole (PBH) production in radiation era. The dependence of the produced PBH's mass on model parameters is studied, as well as the form of predicted PBH mass spectra.

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I. INTRODUCTION

Curvaton mechanism which has been suggested ~ 15 years ago [1–5] now is the object of intense study. It is assumed, in the standard implementation of the curvaton model, that not the inflaton field perturbations are responsible for the primordial density fluctuations and for the cosmic microwave background fluctuations, but instead the (isocurvature) perturbations of the curvaton field σ . It is assumed that this curvaton field is subdominant during inflation but in post-inflationary epoch when Hubble constant becomes small, $H \sim m$ (where m is the curvaton mass), curvaton starts oscillating in its potential and behaves as nonrelativistic matter. The energy density of the curvaton decreases as $\sim a^{-3}$ (a is the scale factor) whereas the energy density of radiation produced by the inflaton decay decreases as a^{-4} . As a result the curvaton energy density grows relative to radiation energy density until the curvaton contribution becomes significant. If it happens before the curvaton decay one can say that curvaton mechanism is “effective”, in a sense that just the curvaton (rather than inflaton) field perturbations during inflation determine the resulting (adiabatic) curvature perturbations at cosmological scales.

In scenarios with the “effective” curvaton there is the strong constraint on a value of the curvaton mass: it must be much smaller than the Hubble constant during inflation, H_i , otherwise the primordial density perturbations have too large spectral tilt. Moreover, if the ratio m^2/H_i^2 is not small, the coherent length of the curvaton field (i.e., the characteristic size of the region inside of which the field is approximately homogenous) is also too small and, in particular, smaller than the current horizon size. In the latter case, the primordial perturbation spectrum is strongly non-Gaussian, in contradiction with observations.

The condition $m^2/H_i^2 \ll 1$ is too restrictive and prohibits an use, for a description of the curvaton, particle physics models predicting large ratios m^2/H_i^2 at inflation (e.g., some variants of supersymmetric theories). In this connection it is reasonable to consider also the mixed curvaton-inflaton scenarios [6, 7] in which the curvaton perturbations are additional to the usual perturbations produced by the inflaton. Combining two contributions, one can obtain the primordial perturbation spectrum which is in agreement with data at cosmological scales. At the same time, the prediction for smaller scales may be quite unusual: the spectrum can be, e.g., very blue (i.e., the spectral tilt is large and positive) and, besides, the perturbations can be strongly non-Gaussian. In particular, large value of the tilt arises due to non-renormalizable and supergravity corrections to the Lagrangian of some supersymmetric theories inducing mass terms of the order H^2 [8–12].

In most curvaton scenarios it is assumed that the curvaton field in the Universe is highly homogeneous and, as a result, the non-Gaussianity is small. Instead, we assume that, after the long inflationary expansion, the average value of the curvaton field is close to zero, and the local value of the field has a Gaussian probability distribution, variance of which is given by the formula [13, 14]

$$\langle \sigma^2(\mathbf{x}) \rangle = \frac{3H_i^4}{8\pi^2 m_*^2} = \left(\frac{H_i}{2\pi} \right)^2 \frac{1}{t_\sigma}. \quad (1)$$

Here, m_* is the effective curvaton mass which differs from the true curvaton mass m [15]. The corresponding coherent

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length is

$$\ell_c \sim \frac{1}{H_i} \exp\left(\frac{3H_i^2}{2m_*^2}\right) = \frac{1}{H_i} \exp\left(\frac{1}{t_\sigma}\right). \quad (2)$$

In Eqs. (1) and (2), t_σ is the spectral tilt of the perturbation spectrum of the curvaton field, $t_\sigma = d \ln \mathcal{P}_\sigma / d \ln k$. The assumption that $\bar{\sigma} = 0$ will have real sense if the scale of interest, $\ell_R = a_i/k_R$, will be larger than ℓ_c (both scales are calculated at the end of inflation). In our case, ℓ_R is given by

$$\ell_R = \frac{a_i}{k_{end}} e^N. \quad (3)$$

Here, k_{end} is the scale leaving the horizon at the end of inflation, a_i is the scale factor at the end of inflation (and at the beginning of radiation era), N is a number of e-folds after the scale k_R leaves the horizon. The condition $\ell_R \gg \ell_c$ leads to the inequality $N \gg 1/t_\sigma$. It means that if t_σ is not small ($t_\sigma \sim 1$), and the coherent length ℓ_c is small, one anticipates the blue curvature spectrum (the curvaton contribution) and large non-Gaussianity *at small scales*. In this case, the data at cosmological scales are described by the inflaton fluctuations only.

Traditionally, predictions for the curvature perturbation spectrum in a region of small scales can be constrained with a help of primordial black holes (PBHs), because a large amplitude of the spectrum can lead to their production at radiation era. In the concrete case of the curvaton model, the idea was suggested in [16] and was considered, in more detail, in [17]. More recently, the PBH formation in an axion-like curvaton model was studied in [18].

In the present work we consider the PBH formation and PBH constraints in a special case when adiabatic perturbations at small scales are produced by curvaton only, resulting in a blue curvature spectrum and large non-Gaussianity. It means that in our case the typical size of the “curvaton domain” [19] is relatively small, it is smaller than the horizon size at the moment of the formation of PBH with a given mass.

The plan of the paper is as follows. In the next Section we derive the basic formula for the curvature perturbation spectrum used in the concrete calculations. In Sec. III, the process of PBH production in our curvaton model is considered. The last Section contains the results of the calculation and conclusions.

II. CURVATURE PERTURBATION SPECTRUM FORMULA

The calculation of the curvature perturbation spectrum from the curvaton, $\mathcal{P}_{\zeta_\sigma}$, is based on the expression for the nonlinear curvature perturbation on uniform density hypersurfaces [20, 21],

$$\zeta(t, \mathbf{x}) = \delta N(t, \mathbf{x}) + \frac{1}{3} \int_{\bar{\rho}(t)}^{\rho(t, \mathbf{x})} \frac{d\tilde{\rho}}{\tilde{p} + \tilde{\rho}}, \quad (4)$$

where δN is the perturbed expansion, ρ and p are the local density and the local pressure. From Eq. (4) we have formulas for the local curvaton and radiation densities on hypersurface on which curvaton decays (we use the sudden-decay approximation):

$$\zeta_r = \zeta + \frac{1}{4} \ln \frac{\rho_r}{\bar{\rho}_r}, \quad (5)$$

$$\zeta_\sigma = \zeta + \frac{1}{3} \ln \frac{\rho_\sigma}{\bar{\rho}_\sigma}. \quad (6)$$

Here, ζ is the total curvature perturbation after the decay. Besides, one has the expression for the local curvaton energy density on the hypersurface characterized by ζ_r , once the curvaton starts to oscillate

$$\rho_\sigma = \bar{\rho}_\sigma e^{3(\zeta_\sigma - \zeta_r)}. \quad (7)$$

Using (7), one can connect these perturbations with the field fluctuations during inflation. At the beginning of the oscillations one has

$$\bar{\rho}_\sigma e^{3(\zeta_\sigma - \zeta_r)} = \frac{1}{2} m_{\sigma_{osc}}^2, \quad (8)$$

and the connection is given by the relation [21]

$$\sigma_{osc} = g(\sigma_*) = g(\bar{\sigma}_* + \delta\sigma_*). \quad (9)$$

Here we assume that the quantum fluctuations of the curvaton at the horizon exit during inflation are Gaussian, i.e., $\sigma_* = \bar{\sigma}_* + \delta\sigma_*$, with no higher order (non-Gaussian) terms.

We use the expansion [21]

$$\sigma_{osc} = g(\bar{\sigma}_*) + g'\delta\sigma_* + \frac{1}{2}g''(\delta\sigma_*)^2 + \dots, \quad (10)$$

where

$$g(\bar{\sigma}_*) \equiv \bar{g} = \bar{\sigma}_{osc}, \quad g' = \left. \frac{d\sigma_{osc}}{d\sigma_*} \right|_{\sigma_*=\bar{\sigma}_*}, \quad g'' = \left. \frac{d^2\sigma_{osc}}{d\sigma_*^2} \right|_{\sigma_*=\bar{\sigma}_*}. \quad (11)$$

The final expression for ζ_σ is (assuming that $\Omega_\sigma \ll 1$) (see, e.g., [22])

$$\zeta_\sigma \cong \frac{2}{3}R_\sigma \frac{g'}{\bar{g}} \delta\sigma_* + \frac{1}{3}R_\sigma \frac{g'^2}{\bar{g}^2} (\delta\sigma_*)^2, \quad (12)$$

where

$$R_\sigma = \left. \frac{3\Omega_\sigma}{4 - \Omega_\sigma} \right|_{dec}, \quad \Omega_\sigma = \left. \frac{\bar{\rho}_\sigma}{\bar{\rho}_\sigma + \bar{\rho}_r} \right|_{dec} = \left. \frac{\bar{\rho}_\sigma}{\bar{\rho}} \right|_{dec}. \quad (13)$$

The expression for R_σ is given by [3, 23, 24]

$$R_\sigma \cong \frac{\bar{\sigma}_{osc}^2}{M_P^2} \sqrt{\frac{m}{\Gamma_\sigma}} = \frac{\bar{g}^2}{M_P^2} \sqrt{\frac{m}{\Gamma_\sigma}}, \quad (14)$$

where Γ_σ is the decay rate of the curvaton into radiation. It follows from (14) that, if $\bar{\sigma}_{osc}$ goes to zero, the linear term in Eq. (12) vanishes. In this approximation, one has

$$\zeta_\sigma = \frac{1}{3}R_\sigma \frac{g'^2}{\bar{g}^2} (\delta\sigma_*)^2. \quad (15)$$

In what follows, we assume that the curvaton field is frozen during inflation, so we put $\sigma_* \approx \sigma_e$, where σ_e is the value of the field at the end of inflation. The connection of $\delta\sigma_e^2$ -spectrum with the σ_e -spectrum is [16]

$$\mathcal{P}_{\delta\sigma_e^2}^{1/2} = \left(\frac{4}{t_\sigma} \mathcal{P}_{\sigma_e}^2 \right)^{1/2}, \quad (16)$$

and the spectrum of the curvaton field is [16]

$$\mathcal{P}_{\sigma_e} = \left(\frac{H_i}{2\pi} \right)^2 \left(\frac{k}{k_R} \right)^{t_\sigma} = e^{-(N_{infl} - N)t_\sigma} \left(\frac{k}{H_0} \right)^{t_\sigma}. \quad (17)$$

Here, N_{infl} is the number of e-folds after the cosmological scales leave the horizon. The difference $N_{infl} - N$ is the number of “relevant” e-folds [16].

Finally, we obtain for the curvature spectrum the expression

$$\mathcal{P}_{\zeta_\sigma}^{1/2} = \frac{2}{3}R_\sigma \frac{g'^2}{\bar{g}^2} \frac{1}{\sqrt{t_\sigma}} \frac{H_i^2}{(2\pi)^2} \left(\frac{k}{k_R} \right)^{t_\sigma}, \quad \frac{1}{\bar{g}^2}R_\sigma \approx \frac{1}{M_P^2} \sqrt{\frac{m}{\Gamma_\sigma}}. \quad (18)$$

III. PBH PRODUCTION IN THE CURVATON MODEL

We consider the model with the potential (see, e.g., the recent work [25], where other examples of supergravity potentials are considered):

$$V(\sigma) = \frac{\sigma^2}{2} (m^2 + \alpha H^2(t)). \quad (19)$$

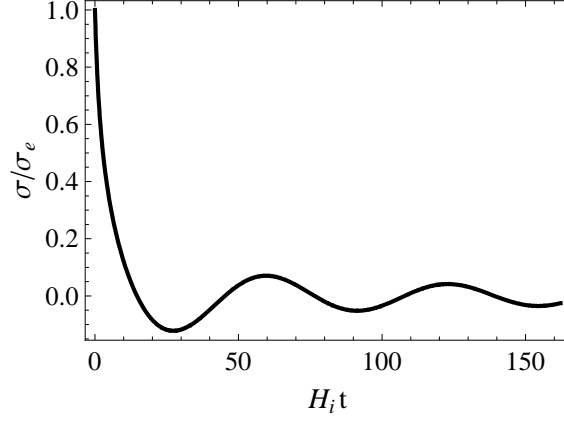


FIG. 1: The solution of Eq. (20) for $\sigma(t)$, for the following set of parameters: $H_i = 10^{13}$ GeV, $m = 0.1H_i$, $\Gamma_\sigma/m = 10^{-21.8}$, the resulting $\sigma_{osc}/\sigma_e = 0.61$.

The equation for the curvaton field σ is

$$\ddot{\sigma} + 3H(t)\dot{\sigma} + V'_\sigma = 0. \quad (20)$$

The solution of this equation for the particular set of parameters is shown in Fig. 1. The calculation starts at moment $t = 0$ corresponding to the end of inflation and the beginning of the radiation-dominated era (the reheating is assumed to be instant). During inflation, at $t < 0$, the solution of Eq. (20) is $\sigma^2 \sim \exp(-t_\sigma H_i t)$ [16], and we use it to obtain the initial condition for (20) at $t = 0$.

The spectral tilt t_σ is simply connected with α (if $m^2 \ll H_i^2$):

$$t_\sigma = \frac{2m_{\sigma*}^2}{3H_i^2} = \frac{2}{3} \left(\alpha + \frac{m^2}{H_i^2} \right) \approx \frac{2}{3} \alpha. \quad (21)$$

In this work we use, for concrete calculations, three values of α , $\alpha = 1$, $\alpha = 0.6$ and $\alpha = 0.38$, so that the curvature spectrum is, in all these cases, blue, with the spectral index $n = 1 + 2t_\sigma$.

The derivative g' is calculated numerically, from the solution of Eq. (20). In our case, because the potential (19) is quadratic, $\sigma_{osc} \sim \sigma_e$, so $g' = \sigma_{osc}/\sigma_e$. For the value of σ_{osc} , we take $\sigma_{osc} \equiv \sigma(t_{osc})$, with the moment of time when oscillations start, t_{osc} , which is determined by the condition [27] (the first root of this Equation in the interval $t > 0$ is used)

$$\frac{\dot{\sigma}(t_{osc})}{H(t_{osc})\sigma(t_{osc})} = 1. \quad (22)$$

We are interested in the calculation of the PBH mass spectrum that are produced from the cosmological perturbations given by the spectrum of Eq. (18). The general formula for the mass spectrum, in the Press-Schechter [26] formalism, is [28]

$$n_{BH}(M_{BH}) = \left(\frac{4\pi}{3} \right)^{-1/3} \left| \frac{\partial P}{\partial R} \right| \frac{f_h \rho_i^{2/3} M_i^{1/3}}{a_i M_{BH}^2}. \quad (23)$$

Here, ρ_i, a_i and M_i are initial (at time of reheating) energy density, scale factor and horizon mass, correspondingly, f_h determines the relation between the produced PBH mass M_{BH} and horizon mass at this moment of time, $M_h(t)$ (we assume in this work that $f_h \approx (1/3)^{1/2} = \text{const}$):

$$M_{BH} = f_h M_h(t), \quad (24)$$

and $P(R)$ is the probability that in the region of comoving size R the smoothed value of curvature perturbation ζ will be larger than the PBH formation threshold value ζ_c :

$$P(R) = \int_{\zeta_c}^{\infty} p_\zeta(\zeta) d\zeta. \quad (25)$$

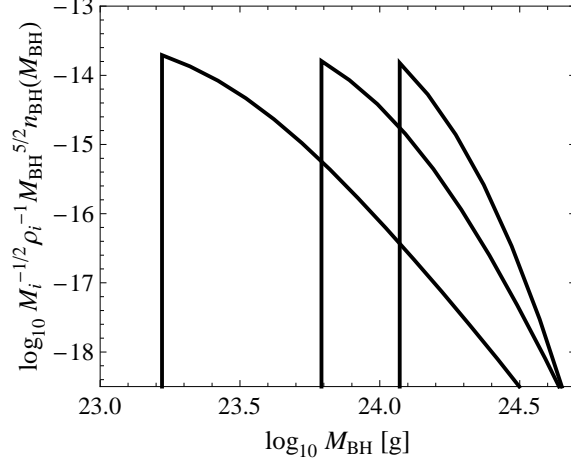


FIG. 2: PBH mass spectra calculation examples. For curves from left to right: $\alpha = 0.38, \Gamma_\sigma/m = 10^{-21.65}$; $\alpha = 0.6, \Gamma_\sigma/m = 10^{-22.22}$; $\alpha = 1, \Gamma_\sigma/m = 10^{-22.5}$. For all cases, $H_i = 10^{13}$ GeV, $m = 0.1H_i$, $\zeta_c = 0.75$.

In our case, when perturbations of ζ are quadratic in σ , one may write

$$\zeta = A(\sigma^2 - \langle \sigma^2 \rangle), \quad A > 0, \quad (26)$$

so

$$p_\zeta(\zeta) = \frac{1}{\sqrt{2\pi\zeta_{min}(\zeta_{min} - \zeta)}} e^{\frac{\zeta - \zeta_{min}}{2\zeta_{min}}}, \quad \zeta_{min} \equiv -A\langle \sigma^2 \rangle, \quad (27)$$

and

$$\zeta_{min} = \zeta_{min}(R) = \left[\frac{1}{2} \int \mathcal{P}_\zeta(k) W^2(kR) \frac{dk}{k} \right]^{1/2}, \quad (28)$$

where $W(kR)$ is the Fourier transform of the window function, and we use a Gaussian one, $W^2(kR) = \exp(-k^2 R^2)$, in this work.

IV. RESULTS AND DISCUSSION

The examples of the PBH mass spectra calculations are shown in Fig. 2. On the vertical axis, we show the combination of parameters giving, approximately, the value of β_{PBH} (energy density fraction of the Universe contained in PBHs at the time of their formation) [28]:

$$\beta_{PBH} \approx \frac{1}{\rho_i} \left(\frac{M_h}{M_i} \right)^{1/2} \int n_{BH} M_{BH}^2 d \ln M_{BH} \approx M_i^{-1/2} \rho_i^{-1} M_{BH}^{5/2} n_{BH}(M_{BH})|_{M_{BH}=M_{BH}^{max}}, \quad (29)$$

where M_{BH}^{max} 's are the values of M_{BH} at maximums of curves in Fig. 2. For the curves shown in Fig. 2, the parameter Γ_σ was tuned so that the three curves corresponding to the different values of α gave approximately the same β_{PBH} .

It is seen from Fig. 2 that for smaller values of α , the mass spectra become more wide. The low mass cut-off of the curves shown is determined by the fact that no PBHs are formed before the curvaton decays at $t = t_{dec}$, so the minimal PBH mass is $M_{BH}^{min} = f_h M_h(t_{dec})$.

For the constraining of the curvaton model parameters, we used the limits for $\beta_{PBH}(M_{BH})$ from the review work [29]. Demanding that PBHs are not overproduced, i.e., the value of $\beta_{PBH}(M_{BH})$ does not exceed the available limits [29], one may obtain the corresponding constraints on the parameters of the considered cosmological model. Such constraints are shown in Fig. 3.

One can see from the resulting Fig. 3 that, generally, PBH constraints are very weak. The forbidden region contains too small values of Γ_σ/m (although the nucleosynthesis limit, $\Gamma_\sigma \gtrsim (1\text{MeV})^2/M_P$, allows such values). The PBH constraint works only for very high values of Hubble constant during inflation, $H_i > 10^{10}$ GeV, and for very large values of curvaton masses, $m \gtrsim (0.001 - 0.1)H_i$. For other values of parameters, the spectrum amplitude, $\mathcal{P}_{\zeta_\sigma}$ is too small and cannot be constrained. One must note also that in the forbidden region the reheating temperatures are rather high ($T_{RH} \sim \sqrt{H_i M_P}$) and, in standard supersymmetric models, gravitinos are overproduced.

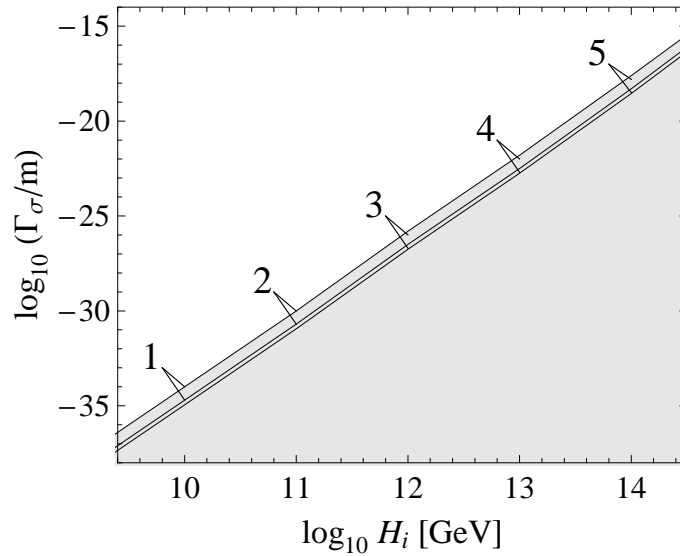


FIG. 3: The resulting constraints on the value of Γ_σ/m for the curvaton model considered in this paper. The shaded regions correspond to the sets of parameters that are prohibited by PBH overproduction. Upper curve corresponds to the case of $m = 0.1H_i$, $\alpha = 0.38$, middle curve - for $m = 0.01H_i$, $\alpha = 0.38$, lower one - for $m = 0.01H_i$, $\alpha = 1$. Numbered labels show the PBH masses that are responsible for obtaining the constraints for the corresponding values of H_i : 1 - $M_{BH} \approx 10^{39}$ g; 2 - $M_{BH} \approx 10^{33}$ g; 3 - $M_{BH} \approx 10^{28}$ g; 4 - $M_{BH} \approx 10^{23}$ g; 5 - $M_{BH} \approx 10^{17}$ g.

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